Spin Transport in Cold Fermi gases: A Pseudogap Interpretation of Spin Diffusion Experiments at Unitarity

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We address recent spin transport experiments in ultracold unitary Fermi gases. We provide a theoretical understanding for how the measured temperature dependence of the spin diffusivity at low T can disagree with the expected behavior of a Fermi liquid (FL) while the spin susceptiblity (following the experimental protocols) is consistent with a Fermi liquid picture. We show that the experimental protocols for extracting χ_s are based on a FL presumption; relaxing this leads to consistency within (but not proof of) a pseudogap-based approach. Our tranport calculations yield insight into the measured strong suppression of the spin diffusion constant at lower T.

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Recent measurements associated with mass transport [1] and spin transport [2] in the ultracold Fermi gases and superfluids are of great interest principally because they provide detailed information about the excitation spectrum, thereby strongly constraining microscopic theories. Equally important are their widespread implications for a host of different strongly correlated systems ranging from quark-gluon plasmas to the high T_c cuprates [3]. Common to these materials are the short mean free paths which lead to "near-perfect fluidity" [1] "bad metallicity" [4] and now bad spin conductivity [2]. These latter are Fermi-gas-based materials with small spin diffusivities D_s that approach the quantum limited value $\hbar/m$ as the temperature is lowered. However, the nature of the excitations in these Fermi gases is currently under debate [5, 6] and a controversy has emerged as to whether the associated normal state of these superfluids is a Fermi liquid or contains an excitation (pseudo)gap. Moreover, there are mixed features in recent spin transport experiments [2] which are indicative of both pairing and Fermi liquid theory.

The goal of this paper is to address these spin transport experiments by Sommer et al. [2] which measure the spin diffusivity D_s, and the spin conductivity σ_s and thereby deduce the spin susceptibility $\chi_s = \sigma_s/D_s$, as functions of temperature at unitarity. Our calculations, based on a pseudogap approach to BCS-BEC crossover theory [7], have the aim of resolving apparent experimental descrepancies between interpretations of χ_s, D_s and σ_s. It should be stressed, however, that we focus on lower T (below the pairing onset T^*) than in Ref. 2 where one can fully contrast Fermi liquid and pseudogap theories. Indeed, if one is to sort out whether the normal state of a unitary gas has a pseudogap or is a Fermi liquid, it is essential to address these spin (as well as mass) transport experiments using multiple theoretical frameworks.

Our key observation is that one must differentiate between the spin susceptibility as computed assuming a constant number of carriers in σ_s and one which is calculated assuming the number of carriers in σ_s increases with T reflecting the presence of a pairing gap. For the latter case one recovers a consistent interpretation of both χ_s and D_s, in contrast to the former Fermi liquid approach, which leads to this mixed interpretation.

The physics of transport in the unitary gas is complicated because, if a pseudogap is present in the normal state, one should accomodate both fermions and fermionic pairs. In recent papers we have included both types of quasi-particles in addressing [8] the observed anomalously low shear viscosity [1] both above and below T_c, as well as magnetic and non-magnetic Bragg scattering [9]. While previous studies of spin transport in the cold Fermi gases have employed, for example, Quantum Monte Carlo, Boltzmann transport and variational approaches [10, 11], we choose to use the Kubo formalism which more readily addresses conservation laws and sum rules. The underlying theory is a BCS-BEC crossover theory in a t-matrix approximation [7]. At $T = 0$, the system is in the BCS-Leggett ground state where there are only superconducting pairs (characterized by the gap Δ_{sc}). For temperatures $0 < T \leq T_c$, there is a comixture of superconducting pairs, noncondensed pairs (characterized by the gap Δ_{pg}), and fermionic quasiparticle excitations.

Essential to satisfying conservation laws and sum rules is the incorporation of Ward identities and collective mode physics. (The latter applies only to the mass transport or "electromagnetic" response.) These issues are

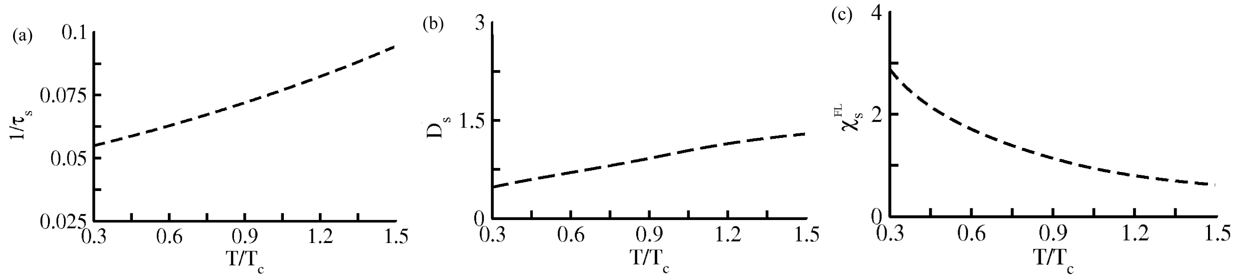

Figure 1: (a) The quasiparticle inverse lifetime τ_s^{-1} in units E_f. The values are inferred from RF experiments [12] and are used as input in our microscopic calculations [8]. The temperature range is limited to $0.3T_c < T < 1.5T_c$ where the RF-derived lifetime data are available. (b) The spin diffusivity D_s for a homogenous unitary Fermi gas in our microscopic theory. It is normalized by its value at T_c. (c) The spin susceptibility χ_s^{FL} computed assuming a temperature independent effective number of carriers and normalized by the value at T_c. At low T, Figs. (a), (b) and (c) can be directly compared to Figs. 2,3 and 4a respectively in Ref. 2.

addressed in Refs. 9, 13. The presence of both condensed and non- condensed pairs in transport leads to the usual Maki-Thompson (MT) and Aslamazov-Larkin (AL) diagrams, which are demonstrably consistent with gauge invariance. A different diagram sub-set necessarily appears in the spin response, as compared with the mass-transport (or electromagnetic response); in the former the AL diagrams are not present, nor do collective mode (or phononic Goldstone bosons) enter [9, 14]. The fermionic self energy on which our work is based has become the *literature standard* for ($T > T_c$) pseudogap theories [15, 16]

$$\Sigma(\mathbf{p}, i\omega_n) = -i\gamma + \frac{\Delta_{pg}^2}{i\omega_n + \xi_{\mathbf{p}} + i\gamma} + \frac{\Delta_{sc}^2}{i\omega_n + \xi_{\mathbf{p}}} \qquad (1)$$

where $\xi_{\mathbf{p}}$ is the free quasiparticle dispersion and γ is the fermionic inverse-lifetime associated with the conversion from fermions to non-condensed pairs. This general form for Σ in Eq(1) has been applied in both experimental[17] and theoretical[18] radio frequency (RF) studies of the cold Fermi gases. In the weak dissipation limit where γ is small, there is little distinction between condensed and non-condensed pairs, while in the strong dissipation limit, this distinction is enforced. In this paper we find that these two approaches tend to converge for s-wave pairing and the lifetime parameters obtained from RF experiments [12]. For reasons of transparency, for the most part the equations we present are in the weak dissipation limit.

We write the key equations first for the dc spin conductivity in the more general strong dissipation limit and second for the the spin-spin correlation function at general $(\mathbf{q}, \omega)$ (called $Q_{00}^s(\mathbf{q}, \omega)$) but in the limit in which the non-condensed pair lifetime is relatively long:

$$\sigma_s = -\lim_{\omega\to 0}\lim_{q\to 0}\frac{1}{6m^2\omega}\mathrm{Im}\sum_P \mathbf{p}^2\Big[G_{P^+}G_P - F_{\mathrm{sc},P^+}F_{\mathrm{sc},P} - F_{\mathrm{pg},P^+}F_{\mathrm{pg},P}\Big]_{i\Omega_l\to\omega^+}, \qquad (2)$$

$$\begin{aligned} Q_{00}^s(\mathbf{q},\omega) = \sum_{\mathbf{p}}\Big[&\frac{E_{\mathbf{p}}^+ + E_{\mathbf{p}}^-}{E_{\mathbf{p}}^+E_{\mathbf{p}}^-}\frac{E_{\mathbf{p}}^+E_{\mathbf{p}}^- - \xi_{\mathbf{p}}^+\xi_{\mathbf{p}}^- - \Delta_{\mathrm{sc}}^2 - \Delta_{\mathrm{pg}}^2}{\omega^2 - (E_{\mathbf{p}}^+ + E_{\mathbf{p}}^-)^2}\big(1 - f(E_{\mathbf{p}}^+) - f(E_{\mathbf{p}}^-)\big) \\ &- \frac{E_{\mathbf{p}}^+ - E_{\mathbf{p}}^-}{E_{\mathbf{p}}^+E_{\mathbf{p}}^-}\frac{E_{\mathbf{p}}^+E_{\mathbf{p}}^- + \xi_{\mathbf{p}}^+\xi_{\mathbf{p}}^- + \Delta_{\mathrm{sc}}^2 + \Delta_{\mathrm{pg}}^2}{\omega^2 - (E_{\mathbf{p}}^+ - E_{\mathbf{p}}^-)^2}\big(f(E_{\mathbf{p}}^+) - f(E_{\mathbf{p}}^-)\big)\Big], \end{aligned} \qquad (3)$$

where $P^+ = (\mathbf{p} + \mathbf{q}, i\omega_n + i\Omega_l)$, $P = (\mathbf{p}, i\omega_n)$, and $\omega^+ = \omega + i0^+$. The quantity $i\omega_n$ ($i\Omega_l$) is a fermionic (bosonic) Matsubara frequency. The excitation energy is $E_{\mathbf{p}} = \sqrt{\xi_{\mathbf{p}}^2 + \Delta^2}$ where $\Delta = \sqrt{\Delta_{sc}^2 + \Delta_{pg}^2}$. The superscripts $\pm$ on $\xi^\pm$ and $E^\pm$ indicate the momentum argument $\mathbf{p} \pm \frac{\mathbf{q}}{2}$. Here f is the Fermi function and $G = G(\Sigma)$ is the dressed Green's function. Note that while one can interpret F_{sc} as the usual Gor'kov Greens function reflecting superconducting order, there must also be a counterpart F_{pg} (discussed in detail elsewhere [3] which reflects non-condensed pairs). Interestingly, in the weak dissipation

limit the spin transport correlation functions depend only on the total pairing gap. When pg effects are dropped, these equations reduce to their usual BCS counterparts.

Any consistent theory of the spin transport must be compared with the f-sum rule

$$\int_{-\infty}^{\infty} d\omega\omega\chi''(\mathbf{q},\omega)/\pi = n\mathbf{q}^2/m \qquad (4)$$

where $\chi''(\mathbf{q},\omega) = -\mathrm{Im}\mathrm{Q}^{\mathrm{s}}_{00}/\pi$ and we have verified from Eq. (3) that this can be proved analytically.

In the weak dissipation limit, it follows from Eq(2) and Ref.9 that the spin conductivity $\sigma_s = -\lim_{\omega\to 0}\lim_{\mathbf{q}\to 0}\frac{\omega}{\mathbf{q}^2}\mathrm{Im}\frac{\mathrm{Q}^{\mathrm{s}}_{00}(\mathbf{q},\omega)}{\pi}$. Importantly, it is common to interpret the spin conductivity in terms of an effective carrier number $\left[\frac{n}{m}(T)\right]_{\mathrm{eff}}$

$$\sigma_s = \left[\frac{n}{m}(T)\right]_{\mathrm{eff}}\tau_s(T) = \frac{2}{3}\tau_s\sum_{\mathbf{p}}\mathbf{v}^2_{\mathbf{p}}\Big(-\frac{\partial f}{\partial E_{\mathbf{p}}}\Big) \qquad (5)$$

where $\tau_s = 1/\gamma$ which can be directly associated with $\frac{1}{\Gamma_{sd}}$, the spin drag relaxation time. Here $\mathbf{v}_{\mathbf{p}} = \partial E_{\mathbf{p}}/\partial\mathbf{p}$. The microscopic Kubo calculation presented here leads to an identification between the spin-drag lifetime, τ_s and the quasi-particle lifetime γ^{-1}, since spin and "charge" or mass are carried by the same quasi-particles. This identification was also observed in previous Kubo calculations of spin diffusion in Helium-3 [19]. That the inter-conversion between fermions and bosons is the physical origin of the lifetime leads us to speculate that τ_s is smallest for the unitary gas [3] where the number of bosonic pairs and fermions are roughly comparable. This appears consistent with the findings in Ref. 2. The spin susceptibility is $\chi^{pg}_s = 2\sum_{\mathbf{p}}\Big(-\frac{\partial f}{\partial E_{\mathbf{p}}}\Big)$. Alternatively, from the small ω and $\mathbf{q}$ hydrodynamics, it is seen that $\chi_s = \sigma_s/D_s$. [20]. This latter approach is used in recent experiments [2].

We now turn to these experiments. In Fig.1(a), the values of τ_s^{-1} inferred from RF data [12] are plotted in units E_F [8]. The lifetime increases for lower temperatures since inter-conversion processes cease when non-condensed bosons disappear; in this way fermions become long lived. At the lowest T considered, this figure shows similar trends to Fig 2 in Ref.2. Here one associates Γ_{sd} with τ_s^{-1}.

The microscopically computed spin diffusivity D_s is shown in Fig.1(b). Importantly we find the spin diffusivity is suppressed at low temperatures reflecting the suppression in the spin conductivity σ_s as a result of the reduced number of carriers. As in experiment, D_s is theoretically found to decrease and (at the lower T) this figure shows similar trends to Fig. 3 in Ref. 2. We estimate D_s at T_c is $0.61\hbar/m$, which when corrected by a factor of 5.3, estimated to account for trap effects [2], is in reasonable agreement. As $T \to 0$ we cannot rule out an upturn in D_s, depending on $\tau_s(T)$.

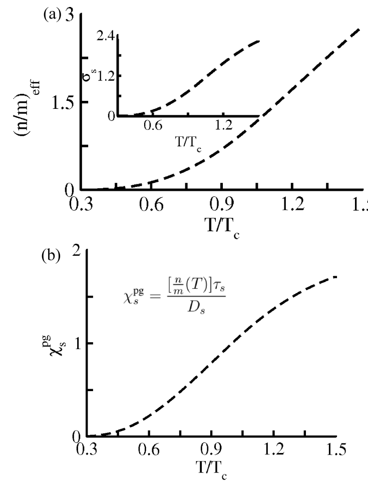

Figure 2: (a) The effective carrier number $(n/m)_{\mathrm{eff}}$ for a homogenous unitary Fermi gas in our microscopic theory normalized by the value at T_c. It vanishes at low temperature because of the pairing gap Δ. (a) Inset:The spin conductivity σ_s for a homogenous unitary Fermi gas in the microscopic theory normalized by the value at T_c. (b) The spin susceptibility χ^{pg}_s for a homogenous unitary Fermi gas in our microscopic theory normalized by the T_c value.

The protocol used to deduce the spin susceptibility in Ref. 2 is based on

$$\chi^{FL}_s = \sigma_s/D_s \equiv \frac{n}{m}\tau_s/D_s \qquad \frac{n}{m} = \text{constant} \qquad (6)$$

We use the superscript FL to emphasize that this analysis builds in a Fermi liquid interpretation by assuming that the carrier number is a constant in temperature and that no excitation gap is present. Qualitatively similar, low T experimental behavior is shown in Fig. 4a of Ref. 2. Nevertheless, this agreement between theory and experiment should not be assumed to support a Fermi-liquid based interpretation– but rather to establish internal consistency. [If the behavior for σ_s is assumed to be Fermi liquid like, the computed χ_s will also be so]. As in experiment, χ^{FL}_s increases for low temperature and appears incompatible with a pairing theory since the spin susceptibility does not vanish for low temperatures.

This will be contrasted with the microscopic calculation of the spin susceptibility, based on σ_s (Eq.(5)), within a pseudogap formulation, where $(n/m)_{\mathrm{eff}}$ is strongly temperature dependent. Plotted in Fig.2(a) is this effective carrier number $(n/m)_{\mathrm{eff}}$ as a function of temperature while the inset indicates the behavior for σ_s, computed from Eq.(2). Both σ_s and $(n/m)_{\mathrm{eff}}$ vanish with decreasing temperature due to the pairing gap.

Our calculation of the spin susceptibility $\chi^{pg}_s = \sigma_s/D_s$ is shown in Fig.2(b). This has the expected temperature dependence associated with a pseudogap. Our key observation here is that one must differentiate between

the spin susceptibility as computed assuming a constant number of carriers in σ_s and one which is calculated assuming the number of carriers in σ_s increases with T reflecting the presence of pairing. For the latter case one recovers a consistent interpretation of both χ_s and D_s, in contrast to the former Fermi liquid approach.

We have not, in this paper proved one way or the other whether there is strong evidence for a pseudogap associated with the experimental results presented in [2]. What we have established is that the indications from the spin susceptiblity, which are used to support Fermi liquid theory are based on Eq. (6) which is associated with a Fermi liquid like behavior. This does not establish the absence of a pseudogap. Other claims for this absence, based on thermodynamics of a balanced unitary gas [5], can be countered by noting that the same thermodynamic power fits can be found for a non-Fermi liquid (for example in BCS theory below T_c). Interpretation of recent experiments [6] which address the low T normal phase associated with a Fermi gas of arbitrary imbalance also fall in this category. The only theoretical phase diagram [21] (of which we aware)– addressing where pseudogap and Fermi liquid phases are stable– predicts this low T phase should be Fermi liquid, as observed [6].

A suggestion for establishing how to rule in or rule out a pseudogap in the normal phase of the unitary gas is discussed in Ref. 22. Essential is that one first confirm the known characteristics of the superfluid phase, such as a suppressed spin susceptibility or entropy, and then establish that these features vary rather smoothly persisting somewhat above the transition. These observations are the counterpart of those first applied to the cuprates and recent cold gas experiments [23] have suggested new techniques for measuring these potential spin susceptibility suppressions.

Of great interest is the relation between spin and mass transport. Because spin and "charge" or mass are carried by the same quasi-particles, even in the presence of a pseudogap [9], spin and mass transport behave similarly. The analysis applied in this paper was used to anticipate that the anomalously low shear viscosity [8] of the normal state should persist down to $T \approx 0$, as now observed [1]. As in Ref. 24 we find the excitation gap is responsible for this behavior. It would similarly explain bad metallicity [4] in the pseudogapped high T_c superconductors [3]. It is striking that experiments from the high T_c community [25] have now tended to focus on the temperature dependence of the effective number of carriers (in the presence of a pseudogap) and note that it will affect transport "because $n_{\rm eff}$ may be changing with T." These commonalities highlight the importance of the ultracold gases as a powerful simulation tool for a wide class of condensed matter systems.

In summary, in this paper we presented a theory of spin transport in a non Fermi liquid (FL) scenario and have shown that these experiments do *not* provide evidence against a pseudogap. Following the experimental protocols for extracting the spin susceptibility we find that χ_s appears FL like, but we emphasize this reflects a FL-starting point (presuming T-independent carrier number in σ_s) in the protocols. As in experiment, at lower T we find that D_s is suppressed. Moreover σ_s is even more so, as both reflect the suppressed carrier number due to pseudogap effects.

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